Applying Combine FAHP-DEA-ANP In Selecting Products

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ABSTRACT

Nowadays, different products grant an incredible collection of functions, and new ones are being added at a breakneck pace. More and more users tend to have the best selection. Therefore, this paper applies two approaches for evaluating and benchmarking products. AHP method, which is a common multi-criteria decision making (MCDM) technique these days. It has been applied to various application areas. DEA method has been used as an appropriate tool to evaluate the relative efficiency of Decision Making Units (DMUs) and attracted more attention mainly because of its robustness.

The prior researches integrate the DEA-based quantitative external method and the ANP-based subjective method. In this paper, we discuss how the hybrid fuzzy analytical hierarchy process and data envelopment analysis with respect to ANP approach applied in a method to identify preferred product selection. Furthermore, the paper compares these approaches by illustrating an example to confirm the presenting model.

Keywords: DEA, ANP, Fuzzy AHP
Abbreviation: DEA, Data envelop analysis; AHP, analytic hierarchy process; ANP, analytic network process;

1. Introduction

AHP is one of the most common methods of multi-criteria decision making technique. AHP spans the boundaries of several academic areas including, engineering, management science and mathematics. The wide applicability is due to its simplicity, ease of use, and great flexibility. According to HO et al. (2010), AHP can be applied to make a consistent decision with respect to multiple qualitative and quantitative criteria. In AHP, the consistency among different pairwise comparison is measured using called consistency verification and it helps avoid making inconsistent decisions due to personal or subjective judgments.

The second most popular individual approach is data envelopment analysis (DEA). DEA has attracted more attention mainly because of its robustness. In the past, it was used to measure the relative efficiencies of homogeneous decision making units (DMUs) based on numerical data only. As the supplier selection problem involves both qualitative and quantitative criteria, DEA has been modified to handle qualitative data, such as amount of know-how transfer (Saen, 2006), service (Seydel, 2006), supplier reputation (Saen, 2007a), and so on. In addition, it can now be used to consider stochastic performance measures (Talluri et al., 2006), and handle imprecise data (Saen, 2007a; Wu et al., 2007).

Jyoti , D.K Banwet and S.G. Deshmukh (Jyoti et al., 2008), used an integrated DEA-AHP to evaluate the performance of national R&D organization. As the product selection problem involves both subjective and objective factors, combined DEA and AHP approach has been modified by FAHP-DEA-ANP techniques in this paper (Sueyoshi et al., 2009).

In the literature, there have been several attempts to combine AHP with DEA (Zhang & Cui, 1999; Sinuany-Stern et al., 2000; Takamura & Tone, 2003; Yang & Kuo, 2003; Ramanathan, 2006; Wang et al., 2008). Ramanathan (2006) embedded DEA into AHP and proposed the DEAHP method.In the DEAHP methodology, DEA is employed initially to generate local weights of alternatives using the values from the AHP judgment matrices and subsequently to aggregate local weights to get overall ones. DEAHP has at least one distinct advantage over AHP because when irrelevant alternative(s) is (are) added or removed, it does not suffer from rank reversal. Although Ramanathan (2006) proved that DEAHP measures the true local weights of consistent judgment matrices, he did not establish his theory for matrices with different levels of inconsistencies. In this paper, DEA integrated with fuzzy AHP technique have been used for both objective and subjective criteria in cell phones selection. In fact, DEA has been used to assess selecting cell phones objective criteria and AHP applied to evaluate this case by subjective criteria and finally it acquired final results by a specific matrix which consider output of both techniques.

The paper is organized in the following order. Section 2 presents MCDM approach and a description of the DEA approach with a concise treatment of its models. Section 3 describes result analysis for the approaches and the decision methodology to apply both subjective and objective factors. Section 4 and 5 provide final results of combine approaches and conclusion.

2. Research methods

2.1 MCDM approaches

Each MCDM approach has some basics stages including criterion set, preference structure, alternative set and performance values and final decision retrieves from performance of alternatives. AHP is an appreciated approach among others MCDM methods that was proposed by Saaty (1980).

2.2 Data envelopment analysis for selecting product

In this section, we briefly introduce DEA and then combine selected DEA model with fuzzy AHP approach.

The basic concept of DEA method involves constructing a non-parametric production frontier based on the actual input-output observations in the sample relative to which efficiency of each bank in the sample is measured (Coelli, 1996). DEA is a mathematical programming methodology. It has been employed successfully for assessing the relative performance of a set of firms, usually called decision-making units (DMU), which use the same inputs to produce the same outputs. Classic DEA models (Cooper et al., 2000) estimate a nonparametric linear piecewise frontier called production frontier which is determined by efficient DMUs to evaluate the relative efficiency of the DMUs.

A DMU with the highest ratio of weighted sum of outputs to the weighted sum of inputs is assumed efficient. Hence, the DEA method maximizes the ratio of weighted sum of outputs to weighted sum of inputs for the DMU under consideration subject to the condition that similar ratios for all DMUs be less than or equal to one.

Thus, a model for calculating the efficiency of the *m*th DMU, which is called the base DMU is formulated as follows (Charnes et al., 1994; Ramanathan, 2003),

DEA *model (CCR model)*

$$\max \frac{\sum_{j=1}^{J} v_{jm} y_{jm}}{\sum_{i=1}^{I} u_{im}\, x_{im}}$$

subject to (1)

$$0 \leq \frac{\sum_{j=1}^{J} v_{jm} y_{jm}}{\sum_{i=1}^{I} u_{im}\, x_{im}} \leq 1; n = 1,2, \ldots., N$$

$$v_{jm} y_{jm} \geq \varepsilon > 0; i = 1,2, \ldots, I; j = 1,2, \ldots, J$$

where the subscript i (i=1,...,I) represents inputs, j (j=1,...,J) represents outputs and n represents the DMUs. The variables u_{im} and v_{jm} are the weights of inputs and outputs called DEA multipliers, respectively. The second subscript m represents the base DMU in which the efficiency is calculated. The symbol ε is an infinitesimal or non-Archimedean constant.

The optimal value of the objective function is the DEA efficiency score assigned to the m^{th} DMU. If the efficiency score is 1, the m^{th} DMU satisfies the necessary condition and this unit is considered as efficient. The point is located on the frontier that envelopes all the data, which is usually called the "efficiency frontier"; otherwise, it is DEA inefficient (Ramanathan, 2007).

If either the denominator or numerator of the ratio is forced to equal to one, then the objective function will become linear, and a linear programming problem can be implemented. For example, by setting the denominator of the ratio equal to one, it is an easy assignment to obtain the following *output maximization* linear programming problem. Note that by setting the numerator equal to one, it is equally possible to produce an *input minimization* linear programming problem:

DEA *model (BCC model)*

$$\max \sum_{j=1}^{J} v_{jm} y_{jm}$$

(2)

subject to

$$\sum_{i=1}^{I} u_{im}\, x_{im} = 1$$

$$\sum_{j=1}^{J} v_{jm} y_{jm} - \sum_{i=1}^{I} u_{im}\, x_{im} \leq 0; n = 1,2, \dots, N$$

$$v_{jm} y_{jm} \geq \varepsilon > 0; i = 1,2, \dots, I; j = 1,2, \dots, J$$

Model 2 is called the "output maximizing multiplier version". This model is implemented to compute the efficiency of just the m^{th} DMU. To get the efficiency scores of all other DMUs, *N* such models must be solved, each for a base DMU (*m* = 1, 2, ..., N). In each model, the constraints are the same while the ratio to be maximized is changed. One of the disadvantages of DEA is that it is likely to provide an efficiency score of 1 for all the DMUs when the number of inputs and outputs are large (Ramanathan, 2003). Hence, to distinguish the efficiency scores of all the DMUs, DEA "super efficiency" scores can be used. The super-efficiency score for a DMU can be computed similarly to the efficiency score using Model 2. The only difference is that the constraint in Model 2 that restricts the efficiency score of the base DMU to less than or equal to 1 is excluded. (Ramanathan, 2009).

3. Results

In this case, we deal with both subjective and objective information; hence, to overcome this problem we use two parallel approaches named, DEA and Fuzzy AHP. As usual, DEA is used to determine the most efficient units based on qualitative and quantitative criteria input and output data. In the process, AHP is also implemented to rank some other qualitative criteria. The following summarizes a step by step procedure of the proposed model of this paper.

Step 1: Select fair criteria for evaluation,

Step 2: DEA approach:

1- Determining input and output,
2- Use ANP to capture the weights of the criteria,
3- Solving DEA problems using models (1) ,

Step 3: FAHP approach:

4- Apply FAHP approach for qualitative data,
5- Derive the priority and weights of each output by FAHP questionnaire,
6- Apply fuzzy preference programming method,

Step 4: Compare DEA and FAHP results

3.1 Fuzzy AHP approach

As mentioned before, in this step we consider some qualitative factors from the past experiences, and apply AHP approach to evaluate cell phones as a product. To determine criteria, we use a questionnaire, designed according to experts and some customers. Fig. 1 shows details of the hierarchy for our criteria for AHP implementation.

For soothing this task for people in responding the questions, we used linguistic variables proposed by Cheng (Cheng 1999). There are several approaches for deriving weight in Fuzzy AHP and we choose Mikhailove's method (Mikhailov 2003) as a suitable method compared with others techniques. Consider the following matrix,

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12k} & \cdots & \tilde{a}_{1nk} \\ \tilde{a}_{21k} & 1 & \cdots & \tilde{a}_{2nk} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1k} & \tilde{a}_{n2k} & \cdots & 1 \end{bmatrix}. \quad (3)$$

First, to derive fuzzy comparision matrix we create matrix of pair wise comparisons $\tilde{A}=\{\tilde{a}_{ijk}\}$ (Eq. 1); where $\tilde{a}_{jik}=\frac{1}{\tilde{a}_{ijk}}=(1/u_{ijk},\ 1/m_{ijk},\ 1/l_{ijk})$; and $\tilde{a}_{ijk}=\{l_{ijk}, m_{ijk}, u_{ijk}\}$ also l_{ijk} and u_{ijk} are the lower and the upper bounds, representing the scope of the fuzziness of the fuzzy number, and m_{ijk} is core of the fuzzy number, corresponding to the maximum degree of membership, equal to one. By using concept of α-*level sets* or α-cuts, for a given α-level, the interval version of the fuzzy preference programming (FPP) method tries to find a crisp priority vector $W=(w_1, w_2, ..., w_n)^T$, which satisfies approximately all interval constraints: $l_{ijk}(\alpha) \precsim \frac{w_i}{w_j} \precsim u_{ijk}(\alpha)$. By solving the weighted GFPP model, the crisp priority vector W and the objective function C which is an indicator for measuring the overall consistency of the judgments are obtained (Xiao-dong, et al).

$$\max\ C=\sum_{k=1}^{r} l_k\lambda$$

subject to

$$d_q\lambda_k + w_i - u_{ij}(\alpha)w_j \le d_q \tag{4}$$

$$d_q\lambda_k - w_i + l_{ij}(\alpha)w_j \le d_q$$

$$\sum_{i=1}^{n} w_i = 1;\ w_i > 0;\quad i=1,\dots,n\ ; \lambda_k > 0\ ; k=1,\dots,r\ ; q=1,\dots 2m\ .$$

When the interval judgments are consistent, the maximum value of C is greater or equal to one. For inconsistent judgments, C takes a value between one and zero, which depends on the degree of inconsistency and the values of the tolerance parametersd_q. In addition l_k demonstrates the weight of decision makers. Here we have some pair wise matrices, which are enforced by GFPP model.

3.2 DEA approach

In the stage, for evaluating cell phones qualities, we passed three main phases for determining criteria, assigning weight and final evaluation by DEA.

First, we suppose some quantities factors related to cell phones specification which illustrated in fig. 1 as the sub criteria.

There are some inner relations among main criteria, which were proposed by some experts. Noteworthy, all of these relations change our hierarchy structure to network structure which has been shown in fig.2. Furthermore, for driving more appropriate weight of criteria we applied ANP application to prioritize criteria according to some experts' decision in pair wise comparison.

Hence, we apply 8 products with their criteria's and used super decision software as suitable tools for calculating weight of criteria, in which the output of software has been demonstrated in table 1. For example, as per the entries of Table 1, product 3 has a higher AHP weight than other product, therefore product 3 is considered superior. Because of this characteristic (i.e. higher is better), the AHP weights are considered as outputs.

Any application of the DEA model requires a priori specification of inputs and outputs. Therefore, we extracted the sub criteria (the specification of products) from fuzzy AHP as the outputs. Then, we suppose cost as an input, with respect to the characteristic of inputs (i.e. lower is better).(Ramanathan, 2007).

4. Final results of the application of fuzzy AHP and DEA

The results, presented in Table 2, have been calculated using the GAMS and decision support system software.

5. Conclusions

The selection of appropriate products is a very important problem for any user. Therefore it requires consideration of a multitude of factors. Integrated FAHP-DEA-ANP model has provided a fair and useful technique to evaluate the selection products, some of which can be subjective, while some can be objective. This technique has provided more practical results in giving the benchmark and inefficient criteria.

References

Cheng, C.H. (1999). Evaluating attack helicopters by AHP based on linguistic variable Weight. *European Journal of Operational Research*, 116, 423–435.

Cooper W.W , Seiford L.M and Tone K,(2000). Data Envelopment Analysis: A Comprehensive Text with Models, Applications. Reference and DEA-Solver Software, *Kluwer Academic Publishers, London*

Ho, W. (2008). Integrated analytic hierarchy process and its applications–a literature review. *European Journal of Operational Research*, 186, 211-128.

Mikhailov, L. (2003). Deriving priorities from fuzzy pair wise comparison judgements, *Fuzzy Sets and Systems*, 134, 365–385.

Raei Nojehdehi, R., Bagherzadeh Valami, H., & Najafi, S.E. (2011). Evaluating the membership degree of the DMU's related to the fuzzy frontier in DEA with fuzzy input-output levels: A geometrical approach. *European Journal of Operational Research*, 1(4), 1-29.

Ramanathan, R. (2006). Data envelopment analysis for weight derivation and aggregation in the analytic hierarchy process. *Computers and Operations Research*, 33, 1289-307.

Ramanathan, R. (2007). Supplier selection problem: integrating DEA with the approaches of total cost of ownership and AHP. *Supply Chain Management: An International Journal*, 12, 258-261.

Saen, R.F. (2007a). Suppliers selection in the presence of both cardinal and ordinal data. *European*

Journal of Operational Research, 183, 741-747.

Saen, R.F. (2007b). A new mathematical approach for supplier selection: accounting for non-homogeneity is important. *Applied Mathematics and Computation*, 185, 84-95.

Seydel, J. (2006). Data envelopment analysis for decision support. *Industrial Management & Data Systems*, 106(1), 81-95.

Sinuany-Stern, Z., Mehrez, A. & Hadad, Y. (2000). An AHP/DEA methodology for ranking decision making units. *International Transactions in Operational Research*, 7, 109-124.

Steuer, R.E., & Na, P. (2003). Multiple criteria decision making combined with finance: a categorized bibliographic study. *European Journal of Operational Research*, 150(3), 496-515.

Takamura, Y., & Tone, K. (2003). A comparative site evaluation study for relocating Japanese government agencies out of Tokyo. *Socio-Economic Planning Sciences*, 37(2), 85-102.

Talluri, S., Narasimhan, R. & Nair, A. (2006). Vendor performance with supply risk: a chance constrained DEA approach. *International Journal of Production Economics*, 100, 212-22.

Saaty, T.L. (1980). *The Analytic Hierarchy Process*: new McGraw-Hill: McGraw-Hill.

Vaidya, O.S., & Kumar, S. (2006). Analytic hierarchy process: an overview of applications. *European Journal of Operational Research*, 169(1), 1-29.

Wu, T., Shunk, D., Blackhurst, J. & Appalla, R. (2007). AIDEA: a methodology for supplier evaluation and selection in a supplier-based manufacturing environment. *International Journal of Manufacturing Technology and Management*, 11(2), 174-192.

Wang, Y.-M., Liu, J., & Elhag, T.M.S. (2008), An integrated AHP-DEA methodology for bridge risk assessment. *Computers and Industrial Engineering*, 54, 513-25.

Feng, X.D., & Jiang, J.H. (2009). Study on the ANP&DEAs Quantitative Evaluation Method for the Product Innovation Supplier in Supply Chain, in World Congress on Computer Science and Information Engineering.

Yang, T., & Kuo, C. (2003). A hierarchical AHP/DEA methodology for the facilities layout design problem. *European Journal of Operational Research*, 147(1), 128-136.

Zhang, X.S., & Cui, J.C. (1999). A project evaluation system in the state economic information system of China: an operations research practice in public sectors. *International Transactions in Operational Research*, 6(5), 441-452.

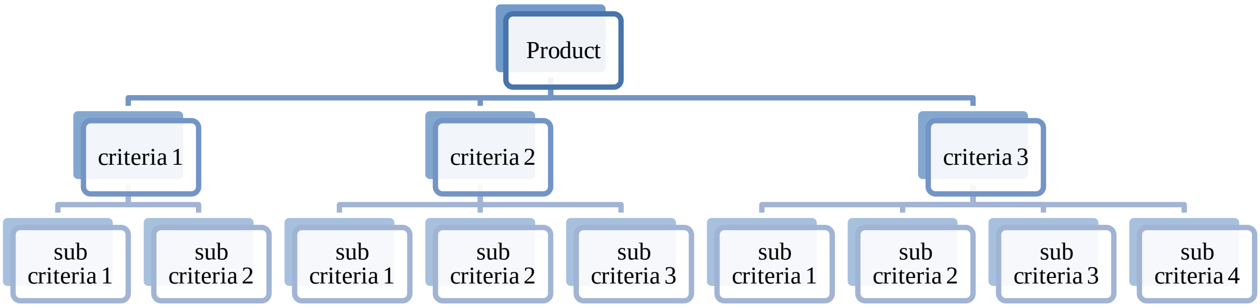

Fig. 1. Hieratical structure of quality model

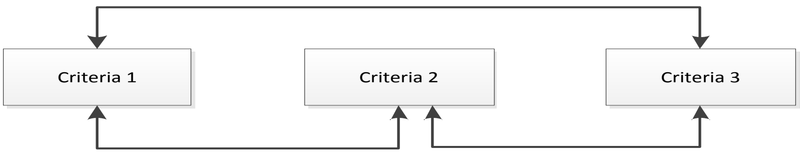

Fig.2. The inner dependence among main criteria

Table 1

The final output priorities of super decision software

Products	P1	P2	P3	P4	P5	P6	P7	P8
Normalizing	0.189	0.271	0.233	0.112	0.190	0.342	0.138	0.103

Table 2

Final decision result

Product	DEA		FAHP	
	rank	Linguistic variable	rank	Linguistic variable
P1	5	Medium	5	Medium
P2	3	High	2	High
P3	2	High	3	High
P4	8	Low	8	Low
P5	4	Medium	4	Medium
P6	1	High	1	High
P7	7	Low	6	Medium
P8	6	Low	7	Low